Math 115

Fall 2017 $\sqrt{xy}$

Lecture 27

Some review:

1) Reduce: $\dfrac{x^4 - 10x^3}{x^2 - 17x + 70} = \dfrac{x^3(x - 10)}{(x-10)(x-7)} = \boxed{\dfrac{x^3}{x-7}}$

2) Simplify: $\dfrac{3x^2 - 5x - 2}{6x^3 + 2x^2 + 3x + 1}$

$= \dfrac{(3x + 1)(x - 2)}{2x^2(3x+1) + 1(3x+1)} = \dfrac{(3x+1)(x-2)}{(3x+1)(2x^2+1)}$

$= \dfrac{x-2}{2x^2+1}$

find all excluded Values:

1) $\dfrac{3x}{7-x}$

$7-x=0$

$7=x$

$\boxed{E.V. : 7}$

2) $\dfrac{2x+3}{x^2+x-2}$ → $x=-2$

$x=1$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$\boxed{\begin{array}{c} E.V. \\ -2 \ \& \ 1 \end{array}}$

3) $\dfrac{whatever}{x^2+5x-14}$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x+7=0 \qquad x-2=0 \qquad$ E.V.

$\boxed{x=-7} \qquad \boxed{x=2} \qquad -7 \ \& \ 2$

4) $\dfrac{whatever}{2x^2+5x-7}$

---

4) $\dfrac{whatever}{2x^2+5x-7}$

$2x^2+5x-7=0$

$a=2 \qquad b=5 \qquad c=-7$

$b^2-4ac=(5)^2-4(2)(-7)$

$=81$

$x=\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-5 \pm \sqrt{81}}{2(2)}=\dfrac{-5\pm9}{4}$

→ $x=\dfrac{-5+9}{4}=\dfrac{4}{4}=\boxed{1}$

→ $x=\dfrac{-5-9}{4}=\dfrac{-14}{4}=\boxed{\dfrac{-7}{2}}$

E.V. $1 \ \& \ \dfrac{-7}{2}$

Multiply: $\dfrac{5x-20}{3x^2+x} \cdot \dfrac{3x^2+13x+4}{x^2-16}$

$= \dfrac{5(x-4)}{x(3x+1)} \cdot \dfrac{(3x+1)(x+4)}{(x-4)(x+4)} = \boxed{\dfrac{5}{x}}$

Divide: $\dfrac{x^2+7x+10}{x^2-x-2} \div \dfrac{x^2+2x-15}{x^2-4x+3}$

$= \dfrac{(x+5)(x+2)}{(x-2)(x+1)} \cdot \dfrac{(x-1)(x-3)}{(x+5)(x-3)}$

$= \boxed{\dfrac{(x+2)(x-1)}{(x-2)(x+1)}}$

Simplify:

1) $\dfrac{5x}{2y} + \dfrac{x}{2y} = \dfrac{5x+x}{2y} = \dfrac{\overset{3}{6x}}{2y} = \boxed{\dfrac{3x}{y}}$

2) $\dfrac{2x}{2x-5} - \dfrac{5}{2x-5} = \dfrac{2x-5}{2x-5} = \boxed{1}$

3) $\dfrac{4x^2+15x}{x+3} - \dfrac{8x+15}{x+3} = \dfrac{4x^2+15x-8x-15}{x+3}$

$= \dfrac{4x^2+7x-15}{x+3} = \dfrac{(x+3)(4x-5)}{x+3} = \boxed{4x-5}$

Simplify : $\dfrac{2x+3}{x^2-x-30} - \dfrac{x-2}{x^2-x-30}$

$= \dfrac{2x+3-x+2}{x^2-x-30} = \dfrac{x+5}{x^2-x-30} = \dfrac{x+5}{(x-6)(x+5)} = \boxed{\dfrac{1}{x-6}}$

Simplify: $\dfrac{9}{y+4} + \dfrac{y-5}{y+4}$

$= \dfrac{9+y-5}{y+4} = \dfrac{y+4}{y+4} = \boxed{1}$

Simplify : $\dfrac{2}{x-3} - \dfrac{1}{x+5}$

$LCD = (x-3)(x+5)$

$= \dfrac{2(x+5)}{(x-3)(x+5)} - \dfrac{1(x-3)}{(x+5)(x-3)}$

$= \dfrac{2(x+5) - 1(x-3)}{(x-3)(x+5)} = \dfrac{2x+10-x+3}{(x-3)(x+5)}$

$= \boxed{\dfrac{x+13}{(x-3)(x+5)}}$

Simplify $\dfrac{4}{x^2-25} - \dfrac{3}{x+5}$

$$= \dfrac{4}{(x+5)(x-5)} - \dfrac{3(x-5)}{(x+5)(x-5)}$$

$$= \dfrac{4-3(x-5)}{(x+5)(x-5)} = \boxed{\dfrac{19-3x}{(x+5)(x-5)}}$$

Simplify

$$\dfrac{x}{x^2-5x+6} - \dfrac{2}{x^2-9}$$

$$= \dfrac{x(x+3)}{(x-3)(x-2)(x+3)} - \dfrac{2(x-2)}{(x-3)(x+3)(x-2)}$$

$$= \dfrac{x(x+3)-2(x-2)}{(x-3)(x-2)(x+3)} = \dfrac{x^2+3x-2x+4}{(x-3)(x-2)(x+3)}$$

$$= \boxed{\dfrac{x^2+x+4}{(x-3)(x-2)(x+3)}}$$

Simplify:

$$\frac{5}{x^2-4} + \frac{2}{x^2+3x+2} - \frac{7}{x^2-x-2}$$

$$= \frac{5(x+1)}{(x+2)(x-2)(x+1)} + \frac{2(x-2)}{(x+2)(x+1)(x-2)} - \frac{7(x+2)}{(x+1)(x-2)(x+2)}$$

$$= \frac{5x+5 +2x-4-7x-14}{(x+2)(x-2)(x+1)} = \boxed{\frac{-13}{(x+2)(x-2)(x+1)}}$$

Solving Rational Equations:

1) find LCD & Excluded Values

2) Multiply by LCD to clear all fractions.

3) Solve the new eqn.

4) Never include excluded Values in your final Solution.

Solve: $\dfrac{x+5}{4} + \dfrac{x+5}{2} = \dfrac{x}{8}$

$LCD = 8$, E.V. : None

$\overset{2}{\cancel{8}} \cdot \dfrac{x+5}{\cancel{4}} + \overset{4}{\cancel{8}} \cdot \dfrac{x+5}{\cancel{2}} = \cancel{8} \cdot \dfrac{x}{\cancel{8}}$

$2(x+5) + 4(x+5) = x$

$2x + 10 + 4x + 20 = x$

$6x + 30 = x$

$6x - x = -30$

$\to 5x = -30$

$\boxed{x = -6}$

$\{-6\}$

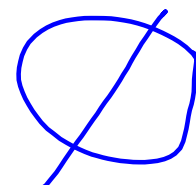Solve $2 + \dfrac{3}{x-3} = \dfrac{x}{x-3}$

$LCD = x-3$, E.V. 3

$(x-3) \cdot 2 + \cancel{(x-3)} \cdot \dfrac{3}{\cancel{x-3}} = \cancel{(x-3)} \cdot \dfrac{x}{\cancel{x-3}}$

$2(x-3) + 3 = x$

$2x - 6 + 3 = x$

$2x - 3 = x$

$2x - x = 3$

$\to x = 3$

$\varnothing$

Solve $\dfrac{2y}{y-2} - \dfrac{4}{y-2} = 4$

LCD $= y-2$, E.V. 2

$(y-2) \cdot \dfrac{2y}{y-2} - (y-2) \cdot \dfrac{4}{y-2} = (y-2) \cdot 4$

$2y - 4 = 4(y-2)$

$2y - 4 = 4y - 8$ → $y = \dfrac{-4}{-2}$ $y = 2$

$2y - 4y = -8 + 4$

$-2y = -4$

$\varnothing$
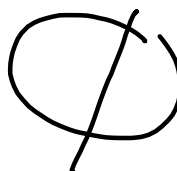
Solve $\dfrac{5}{x-6} = \dfrac{x}{x-2}$

LCD $= (x-6)(x-2)$, E.V.: $6 \text{ } \& 2$

$(x-6)(x-2) \cdot \dfrac{5}{x-6} = (x-6)(x-2) \cdot \dfrac{x}{x-2}$

$5(x-2) = x(x-6)$

$5x - 10 = x^2 - 6x$

$x^2 - 6x - 5x + 10 = 0$

$x^2 - 11x + 10 = 0$

$(x-10)(x-1) = 0$

$\{1, 10\}$ $x = 10$ $x = 1$

Solve: $2 = \dfrac{2x}{x+2} - \dfrac{x-8}{x-2}$

LCD $= (x+2)(x-2)$, E.V.: ± 2

$(x+2)(x-2) \cdot 2 = (x+2)(x-2) \cdot \dfrac{2x}{x+2} -$

$(x+2)(x-2) \cdot \dfrac{x-8}{x-2}$

$2\underbrace{(x+2)(x-2)} = 2x(x-2) - (x+2)(x-8)$

$2(x^2-4) = 2x^2 - 4x - \left[x^2 - 8x + 2x - 16\right]$

$2x^2 - 8 = 2x^2 - 4x - \left[x^2 - 6x - 16\right]$

$2x^2 - 8 = 2x^2 - 4x - x^2 + 6x + 16$

$-8 = -x^2 + 2x + 16$

$-8 + x^2 - 2x - 16 = 0$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$\{-4, 6\}$ $\boxed{x=6}$ $\boxed{x=-4}$

Solve: $\dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{4}{x^2-4}$

LCD $= (x+2)(x-2)$ E.V.: ± 2

$(x+2)(x-2) \cdot \dfrac{1}{x+2} + (x+2)(x-2) \cdot \dfrac{1}{x-2} =$

$(x+2)(x-2) \cdot \dfrac{4}{x^2-4}$

$1(x-2) + 1(x+2) = 4$

$x - 2 + x + 2 = 4$

$2x = 4$ $x = 2$ $\emptyset$

The sum of some number and its reciprocal is $\frac{10}{3}$. Find all such numbers.

Let x be the number

$$x + \frac{1}{x} = \frac{10}{3}$$

Solve $x + \frac{1}{x} = \frac{10}{3}$

$\text{LCD} = 3x$

$\text{E.V.: } 0$

$$3x \cdot x + 3x \cdot \frac{1}{x} = 3x \cdot \frac{10}{3} \quad \rightarrow \quad 3x^2 - 10x + 3 = 0$$

$$3x^2 + 3 = 10x$$

$a = 3 \quad b = -10 \quad c = 3$

$$b^2 - 4ac = (-10)^2 - 4(3)(3) = 64$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-10) \pm \sqrt{64}}{2(3)}$$

$$= \frac{10 \pm 8}{6}$$

$$x = \frac{10 + 8}{6} = \frac{18}{6} = \boxed{3}$$

$$x = \frac{10 - 8}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

$$3 \ \dot{\text{ε}} \ \frac{1}{3}.$$

The differce of some number and its reciprocal is equal to $\frac{3}{2}$. find all such numbers.

$$x - \frac{1}{x} = \frac{3}{2}$$

LCD = $2x$, E.V. : 0

$$2x \cdot x - 2x \cdot \frac{1}{x} = 2x \cdot \frac{3}{2}$$

$$2x^2 - 2 = 3x$$

$$2x^2 - 3x - 2 = 0$$

$a = 2$, $b = -3$, $C = -2$

$$b^2 - 4ac = (-3)^2 - 4(2)(-2)$$
$$= 25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{25}}{4} = \frac{3 \pm 5}{4}$$

$\boxed{x=2}$, $\boxed{x=\frac{-1}{2}}$ $2 \ \& \ \frac{-1}{2}$

---

find two consecutive integers such that the sum of their reciprocals is $\frac{3}{2}$.

x & $x+1$ $\Rightarrow$ $\frac{1}{x}$ & $\frac{1}{x+1}$

$$\frac{1}{x} + \frac{1}{x+1} = \frac{3}{2}$$ Solve for x.